DRAFT SF 298

1. Report Date (dd-mm-yy)	2. Report Type 1996	3. Dates covered (from... to)
4. Title & subtitle A Robust Gain-Scheduling Example User Linear Parameter-Varying Feedback		5a. Contract or Grant # 5b. Program Element #
6. Author(s)		5c. Project # 5d. Task # 5e. Work Unit #
7. Performing Organization Name & Address		8. Performing Organization Report #
9. Sponsoring/Monitoring Agency Name & Address		10. Monitor Acronym 11. Monitor Report #

12. Distribution/Availability Statement
Distribution approved for Public Release, Distribution Unlimited

13. Supplementary Notes TR downloaded from a WWW unrestricted URL.

14. Abstract

19970121 159

15. Subject Terms

Security Classification of			19. Limitation of Abstract	20. # of Pages	21. Responsible Person (Name and Telephone #)
16. Report Unclass	17. Abstract Unclass	18. This Page Unclass	Unlimited		

A Robust Gain-Scheduling Example Using Linear Parameter-Varying Feedback

Mark S. Spillman[*], Paul A. Blue[†], and Siva S. Banda[‡]
USAF Wright Laboratory
WL/FIGC, 2210 Eighth Street, Suite 21
Wright-Patterson AFB, OH 45433-7531
(513) 255-8677

Lawton H. Lee[§]
Dept. of Mechanical Engineering
Univ. of California Berkeley
Berkeley, CA 94720
(510) 642-6152

Abstract

A gain-scheduling approach for uncertain Linear Parameter-Varying (LPV) systems with fixed linear fractional relationships on a parameter set is developed. The approach combines LPV theory based on Linear Matrix Inequalities (LMIs) and μ-synthesis to form a new robust approach for large envelope control design. The new approach is used to design an automatically gain-scheduled pitch-rate controller for the F-16 Variable Stability In-Flight Simulator Test Aircraft (VISTA). The ability of the approach to generate controllers for predicted Level 1 flying qualities is illustrated with a high fidelity nonlinear simulation.

Keywords: Aircraft Control, Automatic Gain Control, H_∞ control, Linear Multivariable Systems, Structured Singular Value

1 Introduction

The challenge of most control problems is to obtain as much performance as possible in the presence of uncertainty due to unmodeled dynamics and plant variations. Unfortunately, performance often must be sacrificed for robustness and vice versa. This typically happens

[*]Email: mspillma@falcon.flight.wpafb.af.mil
[†]Email: blue@falcon.flight.wpafb.af.mil
[‡]Email: banda@falcon.flight.wpafb.af.mil
[§]Supported by AFOSR under a National Defense Science and Engineering Graduate Fellowship. Email: lawton@jagger.me.berkeley.edu.

when the physical plant varies significantly during operation. Under these conditions, the classical designer has been forced to do several point designs and gain-schedule the resulting set of controllers [1].

Several new approaches have emerged in the last few years, however, which may improve the entire gain-scheduling process. These methods explicitly take into account the relationship between real-time parameter variations and performance. Controllers can then be designed for whole ranges of operating conditions.

The purpose of this paper is to demonstrate a new gain-scheduling technique for uncertain LPV systems. The technique presented is a modified version of the LPV method developed by Apkarian and Gahinet [2, 3]. Their original approach applies to systems of the form

$$\begin{aligned}\dot{x}(t) &= A(\theta(t))x(t) + B(\theta(t))u(t) \\ y(t) &= C(\theta(t))x(t) + D(\theta(t))u(t)\end{aligned}$$

where $\theta(t)$ is a vector of measurable time-varying parameters and A, B, C and D are fixed linear fractional functions of θ. In this paper, Apkarian and Gahinet's LPV approach is modified and mixed with techniques used in μ-synthesis [4, 5] to develop a robust synthesis method for uncertain LPV systems of the form

$$\begin{aligned}\dot{x}(t) &= A(\theta(t),\delta)x(t) + B(\theta(t),\delta)u(t) \\ y(t) &= C(\theta(t),\delta)x(t) + D(\theta(t),\delta)u(t)\end{aligned}$$

where δ represents the structured uncertainty of the system and A, B, C and D are fixed linear fractional functions of θ and δ. The new mixed approach is then used to design a pitch-rate controller for the F-16 VISTA.

The remainder of this paper is broken into 3 sections. Section 2 introduces some of the necessary background theory and the mixed LPV/μ approach. In Section 3, the approach is used to design a pitch-rate controller for the F-16 VISTA and the closed-loop system is tested with a high fidelity nonlinear simulation. Section 4 summarizes the results presented in this paper.

The notation used throughout the paper is fairly standard. For real symmetric matrices, X, $X < 0$ stands for "negative definite" and means that all the eigenvalues of X are negative. Similarly, $X > 0$ stands for "positive definite" and means that all the eigenvalues of X are positive. The upper and lower Linear Fractional Transformations (LFTs) of a block partitioned matrix, M, with a constant matrix, N, are defined as

$$\begin{aligned} F_u(M,N) &\equiv M_{22} + M_{21}N(I - M_{11}N)^{-1}M_{12} \\ F_l(M,N) &\equiv M_{11} + M_{12}N(I - M_{22}N)^{-1}M_{21} \end{aligned}$$

respectively. For a positive definite Hermitian matrix, L, $L^{1/2}$ is defined such that $F = L^{1/2}$ satisfies $F^*F = L$. Finally, for an arbitrary matrix, Y, $Ker(Y)$ represents the null space of Y.

2 Gain-Scheduling Uncertain LPV Systems

This section briefly describes gain-scheduling for uncertain LPV systems. The results presented here build upon the results presented in [2, 3], to which the reader is referred for further details and proofs. The basic LPV structure for designing gain-scheduled H_∞ controllers is shown in Figure 1.

The LPV plant is given by the upper LFT

$$F_u\left(P(s), \begin{pmatrix} \Theta & 0 \\ 0 & \Delta \end{pmatrix}\right) \tag{2.1}$$

where $P(s)$ is a known Linear Time Invariant (LTI) plant, $\Theta \triangleq \Theta(t)$ is a time-varying parameter block with the structure

$$\Theta(t) \in \{\text{blockdiag}(\theta_1 I_{r1}, \ldots, \theta_M I_{rM})\} : \theta_i(t) \in C \tag{2.2}$$

and Δ represents a constant uncertainty with the structure

$$\Delta \in \{\text{blockdiag}(\tilde{\Delta}_1, \ldots, \tilde{\Delta}_M)\} : \tilde{\Delta}_i \in C^{s_i \times s_i} \tag{2.3}$$

(Note that the $\tilde{\Delta}_i$'s associated with the performance channels are purely fictitious). It is assumed the maximum singular value of Θ and Δ is less than $1/\gamma$ for some positive scalar, γ. The goal is to find the LTI system, K, such that the gain-scheduled controller given by the lower LFT,

$$F_l(K(s), \Theta) \tag{2.4}$$

ensures the closed-loop system has robust performance.

Since the formulation presented below is based on state-space matrices, $P(s)$ is represented by

$$P(s) = \begin{pmatrix} D_{\theta\theta} & D_{\theta 1} & D_{\theta 2} \\ D_{1\theta} & D_{11} & D_{12} \\ D_{2\theta} & D_{21} & D_{22} \end{pmatrix} + \begin{pmatrix} C_\theta \\ C_1 \\ C_2 \end{pmatrix} (sI - A)^{-1} \begin{pmatrix} B_\theta & B_1 & B_2 \end{pmatrix} \tag{2.5}$$

where the partitioning is conformable to the inputs d_θ, d and u and the outputs e_θ, e and y. The problem dimensions are given by:

$$A \in \mathrm{I\!R}^{n \times n}, D_{\theta\theta} \in \mathrm{I\!R}^{q \times q}, D_{11} \in \mathrm{I\!R}^{p_1 \times p_1}, D_{22} \in \mathrm{I\!R}^{p_2 \times m_2}$$

It is assumed that (A, B_2, C_2) is stabilizable and detectable, $D_{22} = 0$ and either $D_{\theta 2} = 0$ or $D_{2\theta} = 0$. The final assumption ensures that the gain-scheduled controller is causal [2].

In [2, 3], LMI feasibility conditions are presented to construct an LFT controller, K, and a positive definite symmetric scaling, L, given by

$$\begin{pmatrix} L_1 & L_2 \\ L_2^T & L_3 \end{pmatrix} \tag{2.6}$$

with $L_2\Theta = \Theta L_2$ and $L_1, L_3 \in L_\Theta$, where

$$L_\Theta = \{L_i > 0 : L_i\Theta = \Theta L_i, \forall\Theta\} \subset \mathrm{I\!R}^{r \times r} \text{ with } r = \sum_{i=1}^{M} r_i$$

for which the LTI system, $F_l(P_a(s), K(s))$, is internally stable and satisfies

$$\left\| \begin{pmatrix} L^{1/2} & 0 \\ 0 & I \end{pmatrix} F_l(P_a(s), K(s)) \begin{pmatrix} L^{-1/2} & 0 \\ 0 & I \end{pmatrix} \right\|_\infty < \gamma \tag{2.7}$$

Here P_a is given by

$$P_a(s) = \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & 0 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} (sI - A)^{-1} \begin{pmatrix} b_1 & b_2 \end{pmatrix} \tag{2.8}$$

with

$$b_1 = \begin{pmatrix} 0 & B_\theta & B_1 \end{pmatrix}, \; b_2 = \begin{pmatrix} B_2 & 0 \end{pmatrix}$$

$$c_1 = \begin{pmatrix} 0 \\ C_\theta \\ C_1 \end{pmatrix}, \; d_{11} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & D_{\theta\theta} & D_{\theta 1} \\ 0 & D_{1\theta} & D_{11} \end{pmatrix}, \; d_{12} = \begin{pmatrix} 0 & I_q \\ D_{\theta 2} & 0 \\ D_{12} & 0 \end{pmatrix}$$

$$c_2 = \begin{pmatrix} C_2 \\ 0 \end{pmatrix}, \; d_{21} = \begin{pmatrix} 0 & D_{2\theta} & D_{21} \\ I_q & 0 & 0 \end{pmatrix}$$

Equation 2.7 guarantees Equation 2.4 is a γ-suboptimal gain-scheduled H_∞ controller for the nominal LPV plant. The feasibility conditions in [2, 3] form a convex program with portions of the scaling matrices as the unknown matrix variables. In order to find the scalings associated with the minimum γ in Equation 2.7, the LMI feasibility conditions have to be solved iteratively for γ. Alternatively, γ can be minimized subject to the existence of the real symmetric matrices $(R, S) \in \mathbb{R}^{n \times n}$, and $(L_3, J_3) \in L_\Delta$ such that

$$\begin{pmatrix} R & I \\ I & S \end{pmatrix} \geq 0 \tag{2.9}$$

$$\begin{pmatrix} L_3 & I \\ I & J_3 \end{pmatrix} \geq 0 \tag{2.10}$$

$$Z_R + \gamma \begin{pmatrix} W_{R2}^T \tilde{J}_3 W_{R2} & 0 \\ 0 & \tilde{J}_3 \end{pmatrix} < \gamma \begin{pmatrix} W_{R2}^T \tilde{J}_3 W_{R2} & 0 \\ 0 & \tilde{J}_3 \end{pmatrix} \tag{2.11}$$

$$Z_S + \gamma \begin{pmatrix} W_{S2}^T \tilde{L}_3 W_{S2} & 0 \\ 0 & \tilde{L}_3 \end{pmatrix} < \gamma \begin{pmatrix} W_{S2}^T \tilde{L}_3 W_{S2} & 0 \\ 0 & \tilde{L}_3 \end{pmatrix} \tag{2.12}$$

if W_{R2} and W_{S2} have full column rank and

$$Z_R = N_R^T \begin{pmatrix} AR + RA^T & R\hat{C}_1^T & \hat{B}_1 \hat{J}_3 \\ \hat{C}_1 R & -\gamma \hat{J}_3 & \hat{D}_{11} \hat{J}_3 \\ \hat{J}_3 \hat{B}_1^T & \hat{J}_3 \hat{D}_{11}^T & -\gamma \hat{J}_3 \end{pmatrix} N_R \tag{2.13}$$

$$Z_S = N_S^T \begin{pmatrix} A^T S + SA & S\hat{B}_1 & \hat{C}_1^T \hat{L}_3 \\ \hat{B}_1^T S & -\gamma \hat{L}_3 & \hat{D}_{11}^T \hat{L}_3 \\ \hat{L}_3 \hat{C}_1 & \hat{L}_3 \hat{D}_{11} & -\gamma \hat{L}_3 \end{pmatrix} N_R \tag{2.14}$$

$$N_R = \begin{pmatrix} W_{R1} & 0 \\ W_{R2} & 0 \\ 0 & I \end{pmatrix}, \quad \begin{pmatrix} W_{R1} \\ W_{R2} \end{pmatrix} = Ker \begin{pmatrix} B_2^T & D_{\theta 2}^T & D_{12}^T \end{pmatrix} \tag{2.15}$$

$$N_S = \begin{pmatrix} W_{S1} & 0 \\ W_{S2} & 0 \\ 0 & I \end{pmatrix}, \quad \begin{pmatrix} W_{S1} \\ W_{S2} \end{pmatrix} = Ker \begin{pmatrix} C_2 & D_{2\theta} & D_{21} \end{pmatrix} \tag{2.16}$$

$$\hat{B}_1 = \begin{pmatrix} B_\theta & B_1 \end{pmatrix}, \quad \hat{C}_1 = \begin{pmatrix} C_\theta \\ C_1 \end{pmatrix}, \quad \hat{D}_{11} = \begin{pmatrix} D_{\theta\theta} & D_{\theta 1} \\ D_{1\theta} & D_{11} \end{pmatrix} \tag{2.17}$$

$$\hat{L}_3 = \begin{pmatrix} L_3 & 0 \\ 0 & I \end{pmatrix}, \quad \hat{J}_3 = \begin{pmatrix} J_3 & 0 \\ 0 & I \end{pmatrix} \tag{2.18}$$

When W_{R2} and W_{S2} have full column rank, Equations 2.9 - 2.12 form a Generalized Eigenvalue Problem (GEVP) (Note: the matrix sums on the left-hand sides of Equations 2.11 and 2.12 are independant of γ once simplified) which can be solved using LMI-LAB^{TM} [6].

If W_{R2} and W_{S2} do not have full column rank, additional dummy variables can be used to form a slightly modified GEVP [6]. This modified GEVP formulation was used to solve the example problem presented in this paper. Depending on the dimensions of d_θ, d, e_θ and e, and the desired accuracy of γ, however, it may be more efficient to solve the LMI feasibility problem iteratively for γ_{min}. In general, GEVPs require more computational effort than LMI feasibility problems and additional dummy variables increase the computation burden.

Notice from Equation 2.7 that the channels associated with the uncertainty, Δ, are completely unscaled. Adding scaling matrix variables to these channels ruins the convexity of the above problem. The above formulation still has value, however, since the current version of μ-Tools [5] cannot handle repeated scalar blocks or static scaling matrices efficiently. The deficiencies in both the LPV and μ-synthesis approaches motivate a mixed LPV approach which can exploit the structure of Θ and Δ less conservatively while minimizing the structured singular value of the closed-loop system. The following modified "D-K" iteration scheme, dubbed "D-K-D" is proposed.

1. Define a scaling associated with the uncertainty, $L_\Delta(s) \in C^{p_1 \times p_1}$ and set it equal to identity.

2. Form the inner D scaling,

$$D_i \equiv \begin{pmatrix} I & 0 \\ 0 & L_\Delta \end{pmatrix} \tag{2.19}$$

   and the system

$$T_i(s) = D_i F_l(P_a(s), K(s)) D_i^{-1} \tag{2.20}$$

3. Use the GEVP to find an outer D scaling (the first "D"),

$$D_o \equiv \begin{pmatrix} L^{1/2} & 0 \\ 0 & I \end{pmatrix} \tag{2.21}$$

   such that

$$\gamma = \left\| \; D_o T_i D_o^{-1} \; \right\|_\infty \tag{2.22}$$

is minimized.

4. Design an H_∞ controller, K, for the system

$$P_s(s) = \begin{pmatrix} L^{1/2}d_{11}L^{-1/2} & L^{1/2}d_{12} \\ d_{21}L^{-1/2} & 0 \end{pmatrix} + \begin{pmatrix} L^{1/2}c_1 \\ c_2 \end{pmatrix} (sI - A)^{-1} \begin{pmatrix} b_1 L^{-1/2} & b_2 \end{pmatrix}$$

   using either the DGKF [7] central H_∞ controller equations (if the plant is regular), or the H_∞ controller equations available in LMI-LAB^{TM} [6].

5. Form

$$T_o(s) = D_o F_l(P_a(s), K(s)) D_o^{-1} \tag{2.23}$$

   and over a frequency range find scalars, m_θ, and structured matrices, M_Δ, minimizing

$$\bar{\mu} = \left\| \begin{pmatrix} m_\theta I & 0 \\ 0 & M_\Delta \end{pmatrix} T_o \begin{pmatrix} m_\theta I & 0 \\ 0 & M_\Delta \end{pmatrix}^{-1} \right\|_\infty \tag{2.24}$$

6. Normalize M_Δ at each frequency by m_θ and obtain L_Δ by fitting a transfer function to the normalized M_Δ's.

7. Form D_i (the second "D") and T_i and go to Step 3. Stop when no further reduction in Step 5 can be obtained.

Note that the above scheme is not guaranteed to find the global optimum solution, however, it has worked well in several practical examples.

3 A Flight Control Example

3.1 The LPV Model

This section presents an application of the "D-K-D" approach to the longitudinal control of the F-16 VISTA. Only the longitudinal motions of the aircraft are considered. Roll and

yaw angular velocities and the sideslip angle of the aircraft are assumed to be zero. The standard short-period equations of motion [1],

$$\begin{pmatrix} \dot{\alpha} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} Z_\alpha & 1 \\ M_\alpha & M_q \end{pmatrix} \begin{pmatrix} \alpha \\ q \end{pmatrix} + \begin{pmatrix} Z_{\delta_e} \\ M_{\delta_e} \end{pmatrix} \delta_e \tag{3.1}$$

are used for design purposes. In Equation 3.1 α represents the angle-of attack, q is the pitch-rate, and δ_e is the elevator deflection. The effects of the trailing edge flaps on the longitudinal dynamics are ignored. At trimmed level flight the dimensional coefficients, Z_α, M_α, M_q, Z_{δ_e}, and M_{δ_e} depend mainly on Mach number, M, and altitude, h.

The Flight Dynamics Directorate of Wright Laboratory has a high fidelity, six degree of freedom, nonlinear simulation model for the F-16 VISTA [8]. This simulation contains accurate models of the propulsion system, actuators, sensors, disturbances, loadings, atmosphere, equations of motion, sensors, and an aerodynamic database for a large range of Mach numbers, altitudes, and angles of attack. This nonlinear model was trimmed and linearized at

$$\begin{aligned} M &= \begin{bmatrix} 0.35 & 0.45 & 0.55 & 0.65 & 0.75 & 0.85 \end{bmatrix} \qquad (3.2) \\ h &= \begin{bmatrix} 1000\ ft & 5000\ ft & 15000\ ft & 25000\ ft \end{bmatrix}. \end{aligned}$$

The data was then fit with polynomial functions of M and h. The polynomial expressions for the coefficients are:

$$Z_\alpha = (2.20e-1) - (4.10e-7)h - (2.60e+0)M + (5.15e-5)Mh \tag{3.3}$$

$$\begin{aligned} M_\alpha = {} & (1.71e+1) - (8.07e-4)h - (6.84e+1)M + (3.31e-3)Mh \\ & +(5.62e+1)M^2 - (2.92e-3)M^2h \end{aligned} \tag{3.4}$$

$$M_q = -(2.28e-1) + (7.06e-6)h - (2.12e+0)M + (4.86e-5)Mh \tag{3.5}$$

$$Z_{\delta_e} = -(1.38e-3) + (8.75e-8)h - (3.40e-1)M + (7.98e-6)Mh \tag{3.6}$$

$$\begin{aligned} M_{\delta_e} = {} & -(8.16e+0) + (1.73e-4)h + (4.06e+1)M - (8.96e-4)Mh \\ & -(9.93e+1)M^2 + (2.42e-3)M^2h \end{aligned} \tag{3.7}$$

The design region includes the flight conditions,

$$M \in \left[\begin{array}{cc} 0.4 & 0.8 \end{array}\right], \quad h \in \left[\begin{array}{cc} 5000\ ft & 25000\ ft \end{array}\right], \tag{3.8}$$

which covers roughly one third of the VISTA's $1g$ sustained maneuver envelope at military power. Since the LPV approach presented above is based upon the Small Gain Theorem [9] it's convenient to express Equations 3.3–3.7 in terms of the normalized variables

$$\delta M \equiv \frac{M - 0.6}{0.2} \in \left[\begin{array}{cc} -1 & 1 \end{array}\right], \quad \delta h \equiv \frac{h - 15000\ ft}{10000\ ft} \in \left[\begin{array}{cc} -1 & 1 \end{array}\right].$$

Letting O_M and O_h equal the order of the mach and altitude fits, respectively, each normalized polynomial expression can be converted to an LFT expression in terms of

$$\Theta = blockdiag(\delta M I_{nM},\ \ \delta h I_{nh}) \tag{3.9}$$

where $nM = O_M(O_h + 1)$ and $nh = O_h$ by writing the dimensional coefficients as

$$C_k = Q_{22} + Q_{21}(\Theta^{-1} - Q_{11})^{-1} = F_u(Q, \Theta) \tag{3.10}$$

The partitioned matrix Q is equivalent to a realization of a multi-dimensional system, so its "order" (the size of Θ) can be reduced without penalty by removing "uncontrollable and unobservable states" associated with δM and δh in turn. While this approach doesn't guarantee the resulting Θ will have the minimum possible size, it's easy to implement, and works well in practice.

3.2 Problem Setup

The problem is to design a pitch-rate controller for the F-16 VISTA which provides robust command tracking with predicted Level 1 handling qualities [10]. Time domain specifications for the pitch-rate response are shown in Figure 2 and Table 1. Note that the "rise-time" parameter, δt, varies with the true velocity of the aircraft (in ft/sec). In order to achieve Level 1 flying qualities the model matching control structure in Figure 3 is used. The reference model is the second order system,

$$\frac{4^2}{s^2 + 2(0.5)(4)s + 4^2} \tag{3.11}$$

which meets Level 1 flying qualities over the entire design envelope. In keeping with the baseline VISTA control design, one reference model was chosen so that speed of the pitch-rate response is virtually the same over the entire design envelope. The maximum allowable pitch-rate command (before the actuators saturate) varies with flight condition. Alternatively, a parameter-varying reference model could be used in the design.

The design plant in Figure 3 also contains a first order approximation of the actuator dynamics,

$$\frac{20.2}{s+20.2} \tag{3.12}$$

input multiplicative uncertainty, sensor disturbances, and weights on the elevator deflection rate, uncertainty, noise, and pitch-rate error. The various design weightings are shown in Figure 4. The control weighting, W_c, reflects the maximum allowable elevator deflection rate of 70 deg/sec and the noise weighting, W_n, depicts a 5% error expected in the measurements. The performance and uncertainty weightings, W_p and W_u, are chosen as lowpass and highpass filters respectively to provide the standard performance/robustness tradeoff. The exact values for W_p and W_u were selected after several design iterations.

A plot of the maximum singular value of the closed-loop system after Step 5 in the 7 step "*D-K-D*" process for the 1^{st} and 4^{th} iterations is shown in Figure 5. The maximum scaled singular value after the 1^{st} iteration (corresponding to the unmodified LPV approach) is 1.48. After 4 iterations the upper bound to μ converges to 0.973. The 13^{th} order controller found after the 4^{th} iteration was reduced to 7^{th} order by truncating the fastest modes and residualizing the resulting system.

3.3 Nonlinear Simulation

A high-fidelity nonlinear parameter-varying step response of the system is shown in Figure 6. The aircraft is initially perturbed from trimmed, level flight at $M = 0.6$, and $h = 15000\ ft$. The pitch-rate response demonstrates predicted Level 1 flying qualities.

4 Summary

A new LPV/μ design approach was presented and used to design an automatically gain-scheduled pitch-rate controller for the F-16 VISTA. The design was tested with a high fidelity nonlinear simulation and Level 1 flying qualities were predicted from the time responses. The method presented takes advantage of familiar concepts in linear control theory (i.e. LFTs, and structured singular value synthesis) and new convex optimization tools (LMIs). The major disadvantage in the approach is the lack of direct way to incorporate realistic rate limitations on the changing parameters of the system. Overall, however, the new approach is promising and warrants further research.

5 Acknowledgement

The authors would like to thank P. Apkarian and P. Gahinet for their helpful comments.

References

[1] J. H. Blakelock, *Automatic Control of Aircraft and Missles.* John Wiley & Sons, Inc., 1991.

[2] P. Apkarian and P. Gahinet, "A Convex Characterization of Gain-Scheduled H_∞ Controllers," *IEEE Trans. Auto. Control*, vol. 40, pp. 853–864, May 1995.

[3] P. Apkarian and P. Gahinet, "Erratum to 'A Convex Characterization of Gain-Scheduled H_∞ Controllers'," *IEEE Trans. Auto. Control*, vol. 40, p. 1681, Sep. 1995.

[4] J. C. Doyle, "Structured Uncertainty in Control System Design," in *Proceedings of the Conference of Decision and Control*, (Fort Lauderdale, FL), pp. 260–265, Dec. 1985.

[5] G. J. Balas, J. C. Doyle, K. Glover, and A. Packard, *μ-Analysis and Synthesis Toolbox: User's Guide.* The MathWorks Inc., 1991.

[6] P. Gahinet, A. Nemirovskii, A. J. Laub, and M. Chilali, *LMI Control Toolbox.* The MathWorks Inc., 1995.

[7] J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis, "State-Space Solutions to Standard H_2 and H_∞ Control Problems," *IEEE Trans. Auto. Control*, vol. AC-34, pp. 831–847, Aug. 1989.

[8] R. J. Adams and S. S. Banda, "Robust Flight Control Design Using Dynamic Inversion and Structured Singular Value Synthesis," *IEEE Trans. Control Sys. Tech.*, vol. 1, pp. 80–92, Jun. 1993.

[9] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control.* Prentice Hall Inc., 1996.

[10] "Military Standard - Flying Qualities of Piloted Vehicles (MIL-STD-1797A)." 1990.

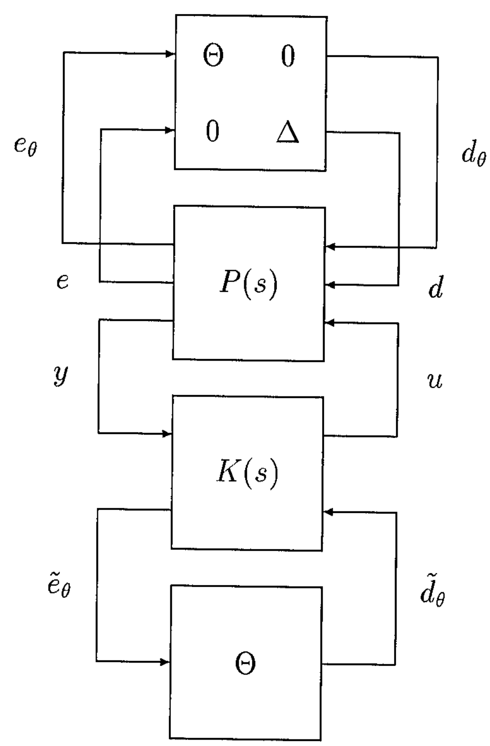

Figure 1: LPV structure

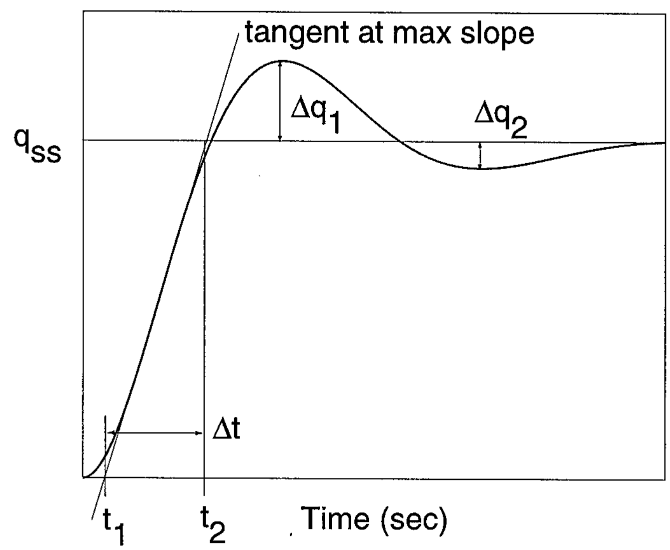

Figure 2: Pitch-rate handling qualities specifications

Table 1: Pitch-rate handling qualities specifications

Parameter	Level I	Level II
t_1 max	0.12 sec	0.17 sec
$\Delta q_2/\Delta q_1$	0.30	0.60
Δt max	$500/V_T$ sec	$1600/V_T$ sec
Δt min	$9/V_T$ sec	$3.2/V_T$ sec

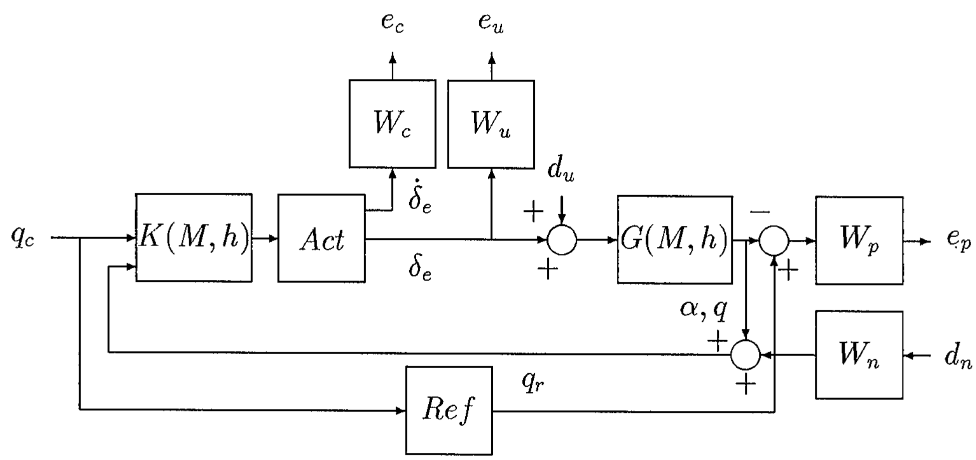

Figure 3: Weighted Plant

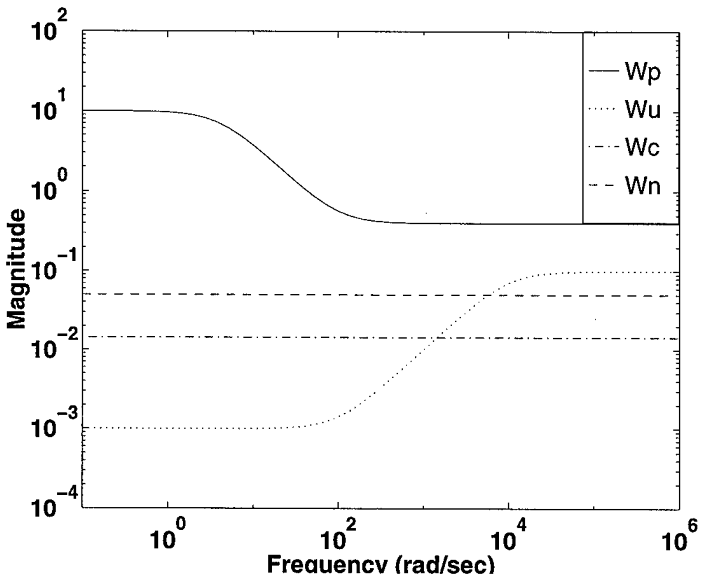

Figure 4: Weighting Functions

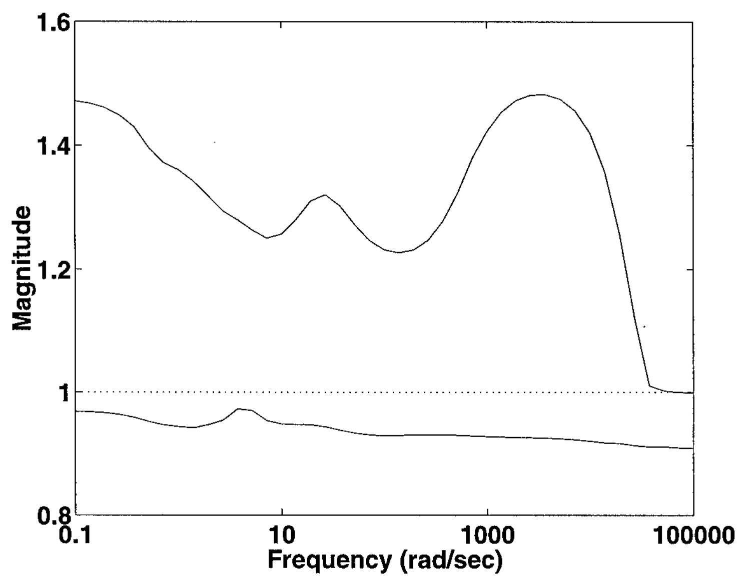

Figure 5: μ upper bound after the 1^{st} and 4^{th} iterations

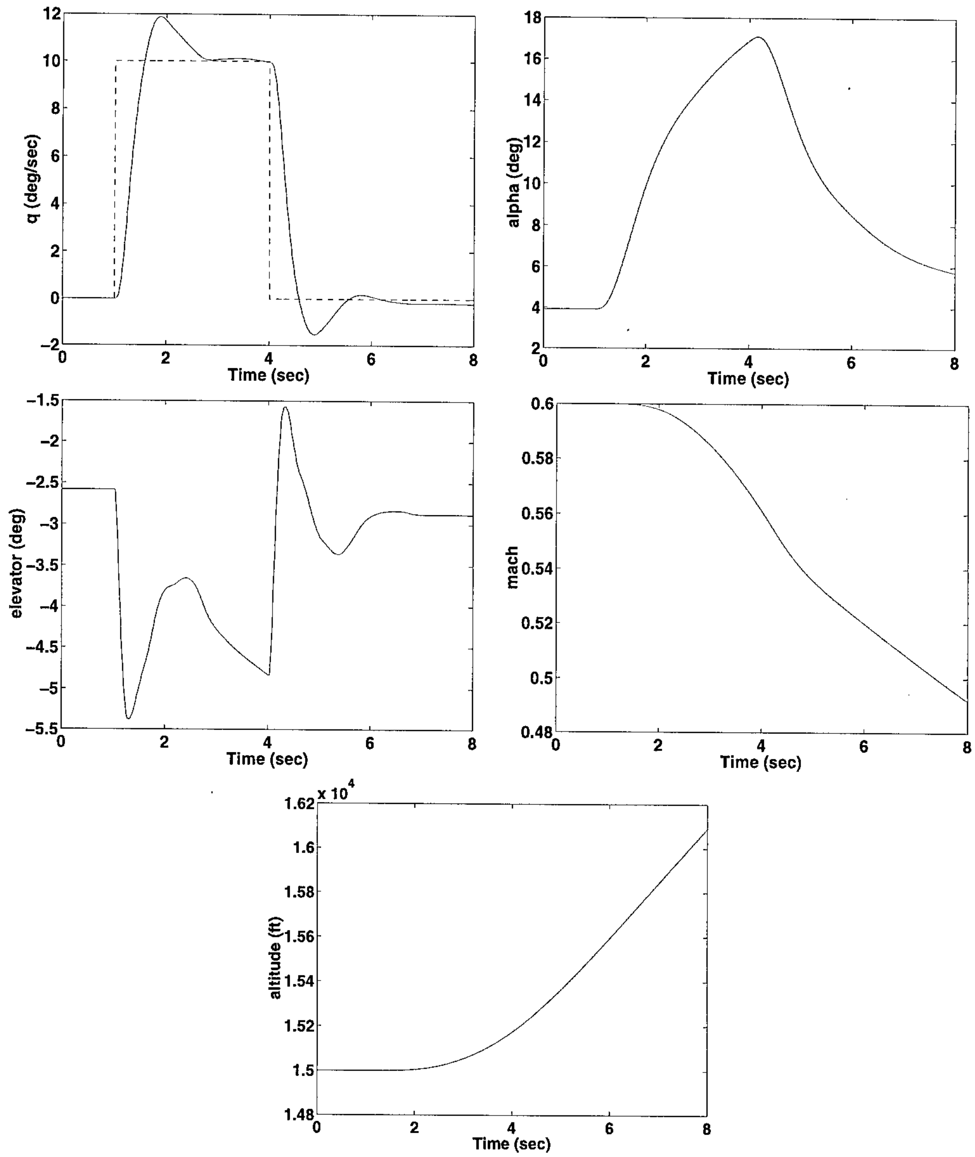

Figure 6: Nonlinear parameter-varying simulation